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# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

**259. Proposed by ARTEMUS MARTIN, M. A., Ph. D., LL. D.; Washington, D. C.**

On page 167 of George Bruce Halsted's *Metrical Geometry* (Mensuration), Boston, 1881, Table of Scalene Triangles, is found the following triangle, viz., Sides 21, 61, 65; Area 420. The sides of a rational scalene triangle, whose sides have no common divisor, can not all be odd; one must be even and the other two odd. It is required to find the error in the sides of the above triangle, assuming that the area is correct.

**I. Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.**

Let $a$, $b$, and $x$ be the three sides of the triangle, and $m$ the area, and we have:

$$\tfrac{1}{2}(a+b+x)\times\tfrac{1}{2}(-a+b+x)\times\tfrac{1}{2}(a-b+x)\times\tfrac{1}{2}(a+b-x)=m^2\text{; or}$$

$$[(a+b)^2-x^2][x^2-(a-b)^2]=(4m)^2;$$

$$x^4-2(a^2+b^2)x^2+(a^2+b^2)^2=4a^2b^2-(4m)^2;$$

$$\text{hence, } x=\pm\sqrt{\{(a^2+b^2)\pm2\sqrt{[(ab+2m)(ab-2m)]}\}}.$$

As $x$ must be a whole number the radical term must be an exact square, which requires the quantity, $(ab+2m)(ab-2m)$ to be an exact square. Now $a$ and $b$ may be: $a=21$, and $b=61$; or $a=21$, and $b=65$; or $a=61$, and $b=65$.

With $a=61$, and $b=65$, but with none of the other values, the inner radical is an exact square. Reducing the outer radical, we have $x=14$, when $m=420$, if we take the lower sign for the inner radical.

**II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

The conditions of the problem require that

$$\tfrac{1}{2}(x+y+z)\tfrac{1}{2}(x+y-z)\tfrac{1}{2}(x-y+z)\tfrac{1}{2}(-x+y+z)=2\times2\times2\times2\times3\times3\times5\times5\times7\times7,$$

$$\text{and } x+y+z=(x+y-z)+(x-y+z)+(-x+y+z).$$

Hence we may have,

$$\left.\begin{array}{r@{\,=\,}l}
x+y+z & 120,\ 140,\ 150,\ \ldots\ldots \\
x+y-z & 70,\ 112,\ 98,\ \ldots\ldots \\
x-y+z & 42,\ 18,\ 48,\ \ldots\ldots \\
-x+y+z & 8,\ 10,\ 4,\ \ldots\ldots
\end{array}\right\}\quad
\begin{array}{l}
x=56,\ 65,\ 73,\ \ldots\ldots \\
y=39,\ 61,\ 51,\ \ldots\ldots \\
z=25,\ 14,\ 26,\ \ldots\ldots
\end{array}$$

Hence, the side given as 21 should be 14.

Also solved in part by the following: **S. A. Corey, Henry Heaton, A. H. Holmes, L. E. Newcomb,** and **J. E. Sanders.**